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ACCUMULATOR FOR ADAPTIVE SIGMA-DELTA MODULATION

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Accumulator for Adaptive Sigma-Delta Modulation

Cross Reference to Related Applications

The present application is a continuation-in-part of U.S. Patent Application
5 10/109,537, filed on March 28, 2002. This application is also a continuation-in-part of
U.S. Patent Application No. 10/357,613, filed on February 4, 2003, which is a
continuation of U.S. Patent Application No. 09/496,756, filed February 3, 2000, which
issued as U.S. Patent 6,535,153 and claims priority from U.S. provisional application
number 60/118,607, filed February 4, 1999. Each of the above-mentioned applications is
10 hereby incorporated herein by reference.

Technical Field

The invention generally relates to signal processing, and more particularly, to
analog to digital conversion using sigma-delta modulation.

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Background Art

Sigma-delta (Σ - Δ) modulation is a widely used and thoroughly investigated
technique for converting an analog signal into a high-frequency digital sequence. See, for
example, "Oversampling Delta-Sigma Data Converters," eds. J. C. Candy and G. C.
Temes, IEEE Press, 1992, (hereinafter Candy) and "Delta-Sigma Data Converters," eds.
20 S. R. Northworthy, R. Schreier, G. C. Temes, IEEE Press, 1997, both of which are hereby
incorporated herein by reference.

In Σ - Δ modulation, a low-resolution quantizer is incorporated within a feedback
loop configuration in which the sampling frequency is much higher than the Nyquist
frequency of the input signal (*i.e.*, much higher than twice the maximum input
25 frequency). In addition, the noise energy introduced in the quantizer is shaped towards
higher frequencies according to a so called "noise-transfer-function" NTF(z), and the
signal passes the modulator more or less unchanged according to a so called "signal-
transfer-function" STF(z).

Fig. 1(a) depicts a simple first order Σ - Δ modulator for a discrete time system
30 having a subtraction stage 101, an accumulator 102 (including an integrator adder 103
and a delay line 104), a one-bit quantizer 105, and a 1-bit digital-to-analog converter

(DAC) 106. In normal operation, an input signal $x(n)$ within the range $[-a, +a]$ is converted to the binary output sequence $y_0(n) \in \pm 1$. Quantizer 105 produces a +1 for a positive input and a -1 for a negative input. The output from quantizer 105 is fed back through DAC 106 and subtracted from input signal $x(n)$ by subtraction stage 101. Thus, the output of subtraction stage 101 represents the difference between input signal $x(n)$ and the quantized output signal $y_0(n)$. As can be seen from Fig. 1(a), the output of accumulator 102 represents the sum of its previous input and its previous output. Thus, depending on whether the output of the accumulator 102 is positive or negative, the one-bit quantizer 105 outputs a +1 or a -1 as appropriate. Herein, and in the appended claims, analog (physical) and digital representations of signals are distinguished from each other by labeling digital one or multi-bit signals with the subscript "0".

In Fig. 1(b), a linear model of Fig. 1(a) is shown, and similarly includes a subtraction stage 107, and an accumulator 111 (including an integrator adder 112 and a delay line 113). Quantizer 105 is replaced by an adder 108 and a noise source 109. To convert signal $y(n)$ to $y_0(n)$, a comparator 110 for detection of the sign of $y(n)$ is included. The basic relationship between the z-transforms of system input $x(n)$, quantizer noise $\gamma_a(n)$, and the two-level output sequence $y(n)$ is:

$$Y(z) = z^{-1}X(z) + (1 - z^{-1})\Gamma_a(z) \quad (1)$$

The signal transfer function and noise-transfer function can be identified as $STF(z) = z^{-1}$ and $NTF(z) = (1 - z^{-1})$, respectively.

Quality of digital representation can be described by the signal-to-noise ratio $SNR = 10 \log_{10} \frac{S}{N}$, where S is the signal power and N is the noise power within a given bandwidth B . Regarding equation (1), the noise power N depends on both the noise transfer function $NTF(z)$ and the overall amount of noise $\Gamma_a(z)$ added in the quantization stage. To improve the SNR, two approaches can be pursued:

(a) for a given overall noise power $\Gamma_a(z)$, i.e., for given quantizer levels $\pm a$, modify the $NTF(z)$ to remove more noise power from the base band by improved noise shaping, and

(b) for a given $NTF(z)$, try to reduce the overall noise power introduced to the system.

Approach (a) can be achieved, for example, by increasing the order of the sigma-delta modulator, as described by Candy. For higher order modulators, the noise transfer function becomes $NTF(z) = (1-z^{-1})^k$, which means an enhanced noise-shaping effect. For examples of approach (b) see Zierhofer C.M., "Adaptive sigma-delta modulation with one-bit quantization," IEEE trans. CAS II, vol. 47, No. 5, May 2000 (hereinafter Zierhofer), and U.S. Patent Application for Adaptive Sigma-delta Modulation with One-bit Quantization, Serial No.: 09/496,756, filed February 3, 2000, which issued as U.S. Patent 6,535,153 (hereinafter U.S. Patent Application Serial No.: 09/496,756), both of which are incorporated herein by reference, where a sigma-delta modulator is employed within a feedback loop configuration. It is shown that the input signal of this modulator applies within a reduced range, and thus the two levels of the quantizer can be reduced. As a consequence, less noise power is introduced to the system, and the SNR is improved.

Summary of the Invention

In connection with developing an adaptive sigma-delta (Σ - Δ) modulator, a new accumulator stage for use in a non-adaptive or adaptive sigma-delta (Σ - Δ) modulator was invented. In accordance with one embodiment of the invention, a system and method for an adaptive sigma-delta (Σ - Δ) modulator includes an input stage that produces a difference signal representing the difference between an analog input signal $x(n)$ and an analog feedback signal $z(n)$. The amplitude of the analog input signal $x(n)$ is within a first range $[-a, +a]$. An accumulator stage produces an accumulated signal that is a function of an accumulation of the difference signal. In particular, producing the accumulated signal may include transforming the accumulation of the difference signal so as to increase average magnitude, while ensuring that an instantaneous magnitude of the accumulated signal does not exceed a predetermined value. A quantization stage produces a quantized digital signal $y_0(n)$ representing the accumulated signal. Based on the quantized digital signal $y_0(n)$, an adaptation stage produces a digital output signal $z_0(n)$, which is converted to the analog feedback signal $z(n)$ by a digital-to-analog converter.

In related embodiments of the invention, the adaptation stage tends to keep the instantaneous magnitude of the analog feedback signal $z(n)$ within the first range $[-a, +a]$ and greater than the analog input signal's $x(n)$ instantaneous magnitude. The accumulator

stage may include an accumulation capacitor, the charge across the capacitor representing an accumulation of the difference signal. The accumulated signal may be based, at least in part, on the voltage across the accumulation capacitor. The capacitance across the accumulation capacitor may be variably controlled such that average magnitude of
 5 voltage across the accumulation capacitor is increased while ensuring instantaneous magnitude of voltage across the accumulation capacitor does not exceed the predetermined value. The accumulation capacitor may be coupled between an input and an output of an operational amplifier.

In further related embodiments of the invention, the adaptation stage may include
 10 a multiplier stage that multiplies the quantized digital signal $y_0(n)$ by a step size $c_0(n)$, and the capacitance across the accumulation capacitor is variably controlled based, at least in part, on the step size $c_0(n)$. The accumulation capacitor may include an array of capacitors, each capacitor in the array capable of being switched so as to vary the capacitance across the accumulation capacitor.

In still further related embodiments of the invention, the digital-to-analog
 15 converter may include an array of weighted capacitors, the array of weighted capacitors capable of acquiring a charge $Q_{DAC}(n)$ negatively proportional to the digital output signal $z_0(n)$. The input sampling stage may include an input sampling capacitor, the input sampling capacitor capable of acquiring a charge $Q_{in}(n)$ proportional to the analog input
 20 signal $x(n)$.

In yet other related embodiments of the invention, the quantized digital signal $y_0(n)$ produced may include a two-level digital output sequence. The two-level digital output may include values of +1 and -1. Producing the digital output signal $z_0(n)$ may include multiplying the quantized digital signal $y_0(n)$ by a step size $c_0(n)$. The step size
 25 $c_0(n)$ may be based on a set Y of code words, where $Y = \{y_0(n), y_0(n-1), y_0(n-2) \dots y_0(n-n_x)\}$, n_x being a predetermined integer. Determining the step size $c_0(n)$ may include increasing the step size $c_0(n)$ if a majority of the code words are equal, or decreasing the step size $c_0(n)$ if the code words alternate. The step size $c_0(n)$ may be non-linear. Multiplying the quantized digital signal $y_0(n)$ by a step size $c_0(n)$ may include using a
 30 look-up-table RAM.

In another embodiment of the invention, a system and method for an adaptive sigma delta modulator includes an input stage that produces a difference signal representing the difference between an analog input signal $x(n)$ and an analog feedback signal $z(n)$. The amplitude of the analog input signal $x(n)$ is within a first range $[-a, +a]$.

An accumulator stage produces an accumulated signal that is a function of an accumulation of the difference signal. The accumulator stage includes an accumulation capacitor having a capacitance that is capable of being variable controlled. The charge of the accumulation capacitor represents the accumulation of the difference signal. A

5 quantization stage produces a quantized digital signal $y_0(n)$ representing the accumulated signal. Based on the quantized digital signal $y_0(n)$ an adaptation stage produces a digital output signal $z_0(n)$. A digital-to-analog converter stage converts the digital output signal $z_0(n)$ to the analog feedback signal $z(n)$.

In related embodiments of the invention, the accumulation capacitor may be
10 variably controlled so as to increase the average magnitude of the voltage across the accumulation capacitor while ensuring an instantaneous magnitude of the voltage across the accumulation capacitor does not exceed a predetermined value. The accumulation capacitor may include an array of capacitors, each capacitor in the array capable of being
15 switched so as to vary the capacitance across the accumulation capacitor. The adaptation stage may include a multiplier stage that multiplies the quantized digital signal $y_0(n)$ by a step size $c_0(n)$, and wherein the capacitance across the accumulation capacitor is variably controlled based, at least in part, on the step size $c_0(n)$. The adaptation stage may tend to keep the instantaneous magnitude of the analog feedback signal $z(n)$ within the first range $[-a, +a]$ and greater than the analog input signal's $x(n)$ instantaneous magnitude.

20 In accordance with another embodiment of the invention, a sigma delta modulator includes an input stage that produces a difference signal representing the difference between an analog input signal $x(n)$ and an analog feedback signal $z(n)$. An accumulator stage produces an accumulated signal that is a function of an accumulation of the difference signal. In particular, the accumulator stage transforms the accumulation of the
25 difference signal so as to increase average magnitude while ensuring instantaneous magnitude does not exceed a predetermined value. A quantization stage produces a quantized digital signal $y_0(n)$ representing the accumulated signal. A digital-to-analog converter stage converts the digital signal $y_0(n)$ to the analog feedback signal $z(n)$.

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Brief Description of the Drawings

The present invention will be more readily understood by reference to the following detailed description taken with the accompanying drawings, in which:

Fig. 1(a) is a block diagram of a prior art first order Σ - Δ modulator for a discrete time system;

Fig. 1(b) is a block diagram of a prior art first order Σ - Δ modulator for a linear model, where the quantizer is replaced by an adder and a noise source;

5 Fig. 2 is a block diagram of an adaptive sigma-delta (Σ - Δ) modulator with one bit quantization that improves the signal-to-noise (SNR) of a Σ - Δ modulator in accordance with one embodiment of the invention;

Fig. 3(a) shows representative waveforms for an adaptive sigma-delta (Σ - Δ) modulator in accordance with one embodiment of the invention;

10 Fig. 3(b) shows representative waveforms for an adaptive sigma-delta (Σ - Δ) modulator in accordance with one embodiment of the invention;

Fig. 4 shows SNR-simulation results representative of various types of analog-to-digital converters;

15 Fig. 5 shows SNR-simulation results representative for adaptive sigma-delta (Σ - Δ) modulators with ideal and non-ideal digital-to-analog converters in accordance with one embodiment of the invention;

Fig. 6 is a block diagram of an adaptive sigma-delta (Σ - Δ) modulator with one bit quantization that includes a high pass filter stage in accordance with one embodiment of the invention;

20 Fig. 7 is a block diagram of an adaptive sigma-delta (Σ - Δ) modulator with one bit quantization having a high pass filter stage that includes a numeric multibit sigma-delta modulator in accordance with one embodiment of the invention;

Fig. 8 shows representative waveforms for the system depicted in Fig. 7;

25 Fig. 9 shows SNR-simulation results pertaining to the signal-to-noise ratio for the system depicted in Fig. 7;

Fig. 10 shows a block diagram of an analog stage of an adaptive sigma-delta modulator, in accordance with one embodiment of the invention; and

Fig. 11 shows a variable accumulation capacitor, in accordance with one embodiment of the invention.

30 Detailed Description of Specific Embodiments

A method and system for an adaptive sigma-delta (Σ - Δ) modulator with one bit quantization that improves the signal-to-noise (SNR) of a Σ - Δ modulator is presented. A

block diagram of the system in accordance with one embodiment of the invention is shown in Fig. 2. The output $y_0(n)$ and two level feedback signal $y(n) \in \pm a$ of a standard Σ - Δ modulator of 1st order is replaced by a multilevel output signal $z_0(n)$ and feedback signal $z(n)$, respectively. The instantaneous magnitude of the multilevel feedback signal $z(n)$ is kept within the range $[-a < z(n) < a]$ and greater than the instantaneous magnitude of the input signal $x(n)$ by an adaptation stage 205. Compared to the non-adaptive modulator, the noise source 107 in Fig. 1(b) is thus reduced, and the SNR is considerably enhanced. The adaptation algorithm of the adaptive Σ - Δ presented can be fully exploited, if the input $x(n)$ is a zero-mean signal, or if the dc-component of $x(n)$ is at least close to zero. If $x(n)$ contains a considerable dc-component, the input dynamic range is reduced. Accordingly, an implementation of a high-pass filter stage is also presented.

Fig. 2 can be separated into an analog and a digital section. An analog input $x(n)$ having range $[-a, +a]$, is inputted into a subtraction stage 208. The analog output of the subtraction stage is then inputted into an accumulator 209 (including an integrator adder 210 and a delay line 211) and then quantized 202. The digital output of the quantizer 202, which may be, but is not limited to, $y_0(n) \in \pm 1$, is multiplied 207 by a step size sequence $c_0(n)$ to create output signal $z_0(n)$. Output signal $z_0(n)$ is passed through a digital-to-analog converter (DAC) 204 to create the analog feedback signal $z(n)$.

Step size sequence $c_0(n)$ is generated in an adaptation stage 205. The step size $c_0(n)$ at a particular instant is controlled by a set of code words $[y_0(n), y_0(n-1), y_0(n-2), \dots]$, which represent the instantaneous value of $y_0(n)$, and a particular (finite) number of previous code words $y_0(n-1), y_0(n-2) \dots$. The primary intention of adaptation stage 205 is to keep the instantaneous magnitude of $z(n)$ greater than the instantaneous magnitude of input signal $x(n)$,

$$|z(n)| > |x(n)|. \quad (2)$$

The way the adaptation stage works is intuitively clear. Step size $c_0(n)$ needs to be increased, if the set $[y_0(n), y_0(n-1), y_0(n-2), \dots]$ contains many equal code words. In this case, $|x(n)|$ tends to exceed $|z(n)|$, which violates condition (2). On the other hand, $c_0(n)$ needs to be decreased, if the set $[y_0(n), y_0(n-1), y_0(n-2), \dots]$ shows an alternating pattern of code words.

Using, for example, a 9-bit DAC 204 for the system shown in Fig. 2, the input range $[-a, +a]$ is subdivided into 511 equally spaced discrete signal levels. Thus, the digital signals $z_0(n)$ and $c_0(n)$ are composed of 9 bits, and 8 bits, respectively. In

accordance with one embodiment of the invention, an adaptation algorithm for a system of 1st order, with constant α chosen to be $\alpha = \frac{16}{15}$, is provided in Tab. 1.

As shown in Tab. 1, the step size is increased by approximately a factor α^3 , if five consecutive code words are equal, and decreased by about a factor α^{-1} , if four consecutive code words have alternating signs. Since step sizes $c_0(n)$ have a limited resolution of 8-bits, the products $\alpha^3 c_0(n-1)$ and $\alpha^{-1} c_0(n-1)$ cannot be implemented exactly, but have to be rounded to the next integer. For small step sizes, the deviations due to rounding are considerable, but this type of imperfection is not essential for the system performance. While a multiplier can be used to calculate $c_0(n)$, in various embodiments of the invention, a look-up-table RAM 206 is utilized instead, where all possible step sizes $c_0(n)$ are stored as 8-bit integers, for example. The minimum and maximum step sizes are then $c_{0,\min} = 1$ and $c_{0,\max} = 255$, respectively. The computation of product $\alpha^3 c_0(n)$ may be achieved by simply increasing the instantaneous RAM-address index by 3. Similarly, for product $\alpha^{-1} c_0(n)$, the instantaneous address index is decreased by 1.

Table 1

Code	Step Size Multiplier
$y_0(n) = y_0(n-1) = y_0(n-2) = y_0(n-3) = y_0(n-4)$	$c_0(n) = \text{round}(\alpha^3 c_0(n-1))$
$y_0(n) = -y_0(n-1) = y_0(n-2) = -y_0(n-3)$	$c_0(n) = \text{round}(\alpha^{-1} c_0(n-1))$
Other combinations	$c_0(n) = c_0(n-1)$

Sample waveforms for an adaptive $\Sigma\text{-}\Delta$ modulator implementing the adaptation algorithm described in Tab. 1 are shown in Figs. 3(a) and 3(b). The first trace 301 in Fig. 3(a) depicts an example of an input signal $x(n)$. The second trace 302 in Fig. 3(a) shows the full wave rectified version $|x(n)|$ together with the magnitude $|z(n)|$ of the DAC-output signal. Condition (2) is fulfilled for almost all samples, but a close examination shows that there is a violation at instant $n = 1643$. This remains singular, however, and has negligible impact on the overall performance of the system. The third trace 303 in Fig. 3(a) illustrates the full DAC-output signal $z(n)$. In Fig. 3(b), the system behavior is shown, if signal $x(n)$ is attenuated by 40dB, i.e. for $x(n)/100$. Traces 304, 305,

and 306 represent an input signal $x(n)$, the full wave rectified version $|x(n)|$ together with the magnitude $|z(n)|$ of the DAC-output signal, and the full DAC-output signal $z(n)$, respectively. As expected, the quantization of signals $|z(n)|$ and $z(n)$ appears more pronounced, and the digital sequence $c_0(n)$ varies between 1 and 3.

5 The examples Figs. 3(a) and 3(b) also demonstrate that the step-size adaptation algorithm works instantaneously, that is, step size multiplier $c(n)$ tracks the individual maxima and minima of input $x(n)$. Signal $c(n)$ can directly be used to estimate the instantaneous power of the input signal, which is advantageous, for example, in signal processing applications for automatic gain control (AGC) for speech signals. Adaptive
10 Σ - Δ modulation schemes typically use comparatively slow adaptation algorithms, where time constants in the range of tens of milliseconds are involved (usually referred to as "syllabic compression"). An example is Chakravarthy, C. V., "An amplitude-controlled adaptive delta sigma modulator," *Radio & Electronic Engineer* (London), vol. 49, pp. 49-54, January 1979, which is hereby incorporated by reference. Systems like this cause
15 gross errors in case of sudden increase of the amplitudes of the input signals and are not practical in signal processing applications, where a permanent accurate representation of the input signal is of importance. Additionally, the use of non-linear step sizes makes the adaptive algorithm more responsive to changes in input amplitudes compared to prior art adaptive algorithms with a constant step size, such as described in Jaggi, M.P.,
20 "Instantaneously Adaptive Delta Sigma Modulator" *Can. Elect. Eng.* 1, Vol. 11 No. 1, 1986, which is herein incorporated by reference. This is important, for example, in keeping the instantaneous magnitude of the first analog feedback signal greater than the input signal's instantaneous magnitude.

In Fig. 4, the SNR of various types of analog-to-digital converters are shown as a
25 function of the input signal's power. The input $x(n)$, within the range $[-1 < x(n) < 1]$ (i.e., $a = 1$), is a periodic zero-mean noise sequence composed of 10000 samples, and the bandwidth is $B = 10\text{kHz}$. Within this bandwidth, amplitudes and phases of the spectral lines are randomized. Different values of signal power are obtained by proportional amplification of this signal. The input power is referenced to the power level of a dc-
30 signal with amplitude $a = 1$. At the maximum input power level shown in Fig. 4 (i.e., at -9.45dB), the maximum signal amplitude reaches 0.99, which is just below the limit of the reference level $a = 1$. The sampling rate for all systems is $\frac{1}{T} = 1\text{MHz}$, and the SNRs are computed within $B = 10\text{kHz}$.

Curve 401 depicts the SNR of an ideal adaptive sigma-delta modulator in accordance with one embodiment of the invention, where the adaptation algorithm of Tab.1 and a 9-bit DAC is used ($a = 1$). Reducing the input power from the maximum level to lower levels, the SNR tends to remain constant. For input levels smaller than about -50dB , the SNR is decreasing. Curve 402 is the SNR of an ideal standard sigma-delta modulator of 1st order with $y_a(n) \in \pm 1$. The maximum SNR is obtained at the maximum input power level, and a decrease of input power results in a decrease of the SNR. Comparing curves 401 and 402 clearly demonstrates the benefit of the adaptive sigma-delta modulator. Curve 402 is very similar to the segment of curve 401 having input levels smaller than about -50dB , shifted to the right by about 48dB . This shift reflects the additional 8 bits of signal $z(n)$ as compared to $y_a(n)$. For input levels smaller than about -50dB , the adaptive sigma-delta modulator operates in a manner similar to a standard sigma-delta modulator, since the feedback-signal $z(n)$ is a two-level signal, $z(n) \in \pm 1/256$. Note that for high input levels, the SNR of the adaptive modulator is not substantially higher than for the standard modulator. However, the input dynamic range has been expanded by approximately 48dB . Curve 403 depicts the SNR of a sigma-delta modulator of 2nd order. Obviously, at lower input levels, the 2nd order system is outperformed by the adaptive modulator. Curves 404 and 405 depict the SNR's of Pulse Code Modulation (PCM) systems with 13 and 14 bit resolutions, respectively. Whereas the 14-bit PCM system is superior to the adaptive sigma-delta modulator for all input levels, the 13-bit PCM system is inferior at least at low-level input signals.

In accordance with one embodiment of the invention, the adaptive sigma-delta modulator includes a multi-bit DAC 204 in the feedback loop, as shown in Fig. 2, since the feedback signal $z(n)$ is a multi-level signal. In preferred embodiments of the invention, the specifications of this DAC 204 is chosen to have minimal effects on the SNR of the adaptive sigma-delta modulator. The non-ideal DAC can be regarded as an ideal converter plus a noise source, with the transfer function of this additional noise source being $-NTF(z) = -z^{-1}$. Thus, this noise (multiplied by -1) is directly added to the input signal and enhances the noise energy in the base band. For the DAC 204, the following assumptions can be made:

- (1) The distribution of DAC-errors is symmetric around zero.
- (2) Each discrete DAC-levels is implemented by means of superposition of binary weights.

(3) Each binary weight itself is composed of elementary unit components. This implies, e.g., that for a particular binary weight which is composed of K unit components, the nominal value is increasing proportional to K , whereas the error is increasing proportional only to \sqrt{K} .

5 For example, for a 9-bit DAC, the pattern $c_0(n) = [1\ 0\ 0\ 1\ 0\ 1\ 0\ 1]$ contains the binary weights 128, 16, 4, and 1, and hence the nominal magnitude of the resulting DAC-level is $\frac{149}{255}a$. However, the single weights can only be realized as $128\left(1 \pm \frac{\Delta}{\sqrt{128}}\right)$, $16\left(1 \pm \frac{\Delta}{\sqrt{16}}\right)$, $4\left(1 \pm \frac{\Delta}{\sqrt{4}}\right)$, and $(1 \pm \Delta)$, which results in a deviation from the nominal value.

10 Fig. 5 shows the SNRs of an adaptive sigma-delta modulator with an ideal 9-bit DAC ($\Delta = 0$) 501, and non-ideal DACs for $\Delta = 1\%$ 502, $\Delta = 3\%$ 503, and $\Delta = 5\%$ 504, where Δ denotes the maximum deviation of the implemented level number $K = 1$ from the nominal value. In all cases, the contribution of the dc-offset error is omitted, and the input signal is the same as for Fig. 4 ($N = 10000$). Fig. 5 shows that the SNR reduction
15 will be within acceptable limits with parameters Δ better than about $\Delta \approx 1\%$.

The adaptive sigma-delta modulator shown in Fig. 2 yields its optimum performance if the input $x(n)$ is a zero-mean signal, or if the dc-component of $x(n)$ is at least close to zero. However, if $x(n)$ contains a considerable dc-component, the input dynamic range is reduced. Consider, for example, an input signal with a dc-component
20 and comparatively very small ac-component. The step size reduction will be governed by the dc-component and not the ac-component, and thus it remains too large. Dc-components can be introduced from either the input signal directly, or by offset-errors due to non-ideal components of the modulator itself. In practical implementations, offset errors in the DAC might occur. In any case, a high-pass filter removes most problems
25 involved with dc- or very low frequency components.

In accordance with one embodiment of the invention, an implementation of a high-pass filter is implemented as shown in Fig. 6. Here, the adaptive sigma-delta modulator 203 of Fig. 2 is embedded in a feedback loop system, which represents a typical digital high-pass filter. The digital output $z_0(n)$ of the modulator is first
30 accumulated 601, resulting in signal $az_0(n)$, and then multiplied 602 by a factor θ , resulting in signal $w_0(n)$. Digital-to-analog conversion of $w_0(n)$ by means of a second

DAC 603 results in signal $w(n)$, which is subtracted 604 from the input $x(n)$. The overall signal-transfer-function $STF_{HP}(z)$ of such a system is given by

$$STF_{HP}(z) = STF(z)HP(z), \quad (3)$$

with the standard Σ - Δ signal-transfer-function, i.e., $STF(z) = z^{-1}$, and the high-pass

5 transfer-function $HP(z) = \frac{1 - z^{-1}}{1 - (1 - \theta)z^{-1}}$. There is a zero at $z = 1$ corresponding to a frequency $f = 0$, and a pole at $z = 1 - \theta$. Using, for example, a sigma-delta rate of 1MHz and $\theta = \frac{1}{1024}$ results in a 3-dB cut off frequency of about 150Hz.

The system shown in Fig. 6 is not well suited for practical implementation, since a 19-bit DAC is involved. Therefore, in accordance with various embodiments of the invention, means to circumvent such a high precision DAC are implemented. In accordance with one embodiment of the invention, inserting a numeric version of a multibit sigma-delta modulator 701 into the system results, for example, in the system as shown in Fig. 7. Assuming $\theta = \frac{1}{1024}$, signal $az_0(n)$ comprises 19 bits. Following the general rules of sigma-delta modulation, this signal is converted to output signal 15 $1024w_{d0}(n)$, which represents the input signal $az_0(n)$, delayed by one clock period. In the present application, signal $w_0(n) = w_{d0}(n+1)$ is used as a high-pass filter feedback signal, which represents the non-delayed input $az_0(n)$, multiplied by $1/1024$. In the present embodiment, two identical 9-bit numeric quantizers 702 and 703 with a transfer characteristics of mid-tread-type are employed. Possible output numbers are -255, -254, ..., -2, -1, 0, 1, 2, ... 254, 255. Note that using the numeric multibit sigma-delta modulator 20 provides a very efficient method to implement the constant-factor multiplication $1/1024$.

Signal $w_0(n)$ is the sigma-delta version of signal $\frac{az_0(n)}{1024}$. Thus, the number of bits has been reduced from 19 in signal $az_0(n)$ to 9 in signal $w_0(n)$. However, following the principles of multibit sigma-delta modulation, although there is a difference of 10 bits 25 both signals contain almost the same information. Information contained in the 10 bit difference is preserved in the temporal fine structure of $w_0(n)$. Additionally, in this example, since both signals $z_0(n)$ and $w_0(n)$ are composed of 9 bits, the 2 DACs of Fig. 6 can be replaced by a single DAC 704, controlled by the sum 705 of signals $z_0(n)$ and $w_0(n)$. Fig. 8 depicts typical waveforms of the system in Fig. 7. The first trace 801 30 shows an input signal $x(n)$ with a step-like transition after 2ms. The second trace 802

depicts the resulting 9-bit signal $w(n)$, which shows the typical low-pass character. With a cut off frequency of about 150Hz, the time is about $\tau = \frac{1}{2\pi 150} \text{ s} \approx 1 \text{ ms}$. As stated above, information is contained in the temporal fine structure of the signal. The third trace 803 shows the difference signal $x(n)-w(n)$, which represents the high-pass filtered version of $x(n)$.

In Fig. 9, the SNR obtained with the system depicted in Fig. 7, using an ideal high-pass filter, is compared with the SNR obtained with the system depicted in Fig. 8, which uses a high-pass filter with numeric multibit sigma-delta modulator (employing 9-bit quantizers of mid-tread type), as waveforms 901 and 902, respectively. The same input signal as specified in Figs. 4 and 5, with $N = 20000$, is used. For both systems, the noise power in the signal band is derived from the difference between the system output signal and a reference signal. The reference signal is the high-pass filtered version of $x(n)$, assuming the ideal transfer function (3). The additional noise introduced by the numeric multibit sigma-delta modulator results in a loss of SNR at low input power levels. The input dynamic range is reduced by approximately 6dB. The decrease of SNR at high input levels is due to a clipping effect and appears, if the sum $w_0(n)+z_0(n)$ exceeds the range $[-255, 255]$. The two SNR-curves 901, 902 are obtained assuming DAC's with $\Delta = 1\%$.

For the analog stages of the adaptive Σ - Δ modulator in Fig.7, standard Switched-Capacitor (SC-) technology may be used, as shown in Fig. 10, in accordance with one embodiment of the system. The 9-bit DAC 1001 is composed of an array of 8 binary weighted capacitors C_{DAC} , $2C_{DAC}$, $4C_{DAC}$, $8C_{DAC}$, $16C_{DAC}$, $32C_{DAC}$, $64C_{DAC}$, and $128C_{DAC}$. The analog input signal is processed by the input sampling stage 1003. Charge- and discharge operations of the arrays 1001 and 1003 are digitally controlled by DAC-Switching control signals 1002 and Input-Switching control signals 1004, which are generated in the digital part of the modulator. Together with the operational amplifier 1006 and the accumulation capacitor 1005, a typical subtract-and-accumulated operation can be described as follows. Each Σ - Δ clock period is subdivided into two sections of roughly equal length, the "sampling-section" and the "accumulation-section". During the "sampling-section", all capacitors in 1001 and 1003 are disconnected from the operational amplifier. The switches at the capacitors are set such that a charge $Q_{in}(n)$ proportional to the input signal $x(n)$ is stored to 1003, and a charge $Q_{DAC}(n)$ proportional to the (negative) overall feedback signal $-[z(n)+w(n)]$ is stored to 1001 (cf. Fig.7). Note that in

practical implementation, input signal $x(n)$ and the overall feedback signal $z(n)+w(n)$ may be referred to a constant potential, such as $V_{ref}/2$. During the "accumulation-section", the ports of the capacitors in 1001 and 1003 are switched such that the capacitors are discharged and the sum of the charges $Q_{in}(n)+Q_{DAC}(n)$ is forced to flow into the accumulation capacitor 1005. Thus, the charge in capacitor 1005 is changed by $Q_{in}(n)+Q_{DAC}(n)$. The sign of the new potential at the output of the operational amplifier referred to $V_{ref}/2$ is sensed by the comparator 1007, and clocked into flip-flop 1008 at the end of the "accumulation-section". Note that for proper operation both, charge-accumulation and the sign-sensing have to be finished within the "accumulation-section", and therefore the response time of the comparator has to be shorter than 50% of a Σ - Δ clock period.

The adaptive Σ - Δ modulator imposes harsh requirements on the comparator 1007. The enhanced input dynamic range causes an equally enhanced dynamic range of the signal at the output of amplifier 1006, which applies at the comparator input. For example, consider a comparator input signal range of a non-adaptive modulator of $\pm 1V$. The corresponding range for an adaptive modulator with a 9-bit DAC is $\pm 1V$ for the largest value of $c(n)$, and $\pm 3.9mV$ for the smallest value of $c(n)$. However, to achieve the desired and theoretically predicted SNR for the $\pm 1V$ -range and $\pm 3.9mV$ range, the switching behavior of the comparator has to be equal for both cases. Unfortunately, comparators tend to become slower as the input signal differences get smaller, and signals in the $\pm 3.9mV$ range are adversely affected. Thus, it has to be ensured that the comparator is sufficiently fast to track very small input signals.

In accordance with one embodiment of the invention, one way to reduce the dynamic range of the comparator input signal is explained with the help of Fig.11. Here, the accumulation capacitor 1005 of Fig.10 is replaced by an SC-array 1101 of capacitors C_{ACC} , C_{ACC} , $2C_{ACC}$, $4C_{ACC}$, $8C_{ACC}$, $16C_{ACC}$, $32C_{ACC}$, $64C_{ACC}$, and $128C_{ACC}$. The size of the feedback capacitor is adapted to the instantaneous input signal power. An overall feedback capacitance $C_{ACC,TOT}(n)$ is realized by switching a particular number of single capacitors in parallel. If the power of the input signal is small, the mean charge across the accumulation capacitor will also be comparatively small. Thus, a smaller overall feedback capacitance $C_{ACC,TOT}(n)$ can be selected, resulting in a larger voltage across the capacitor. On the other hand, an input signal with high power requires a large overall feedback capacitance $C_{ACC,TOT}(n)$ to keep the accumulation signal within specified ranges. The adaptation is carried out in the "sampling-section" of a Σ - Δ clock period, i.e.,

the preparation of charges $Q_{in}(n)$ in 1003 and $Q_{DAC}(n)$ in 1001 is not affected, since 1003 and 1001 are not connected to the operational amplifier. Two cases can be distinguished, (1) an uncharged capacitor is added to the configuration $C_{ACC,TOT}(n)$ of the previous clock period, and (2) a capacitor is removed from the instantaneous configuration $C_{ACC,TOT}(n)$.

- 5 The two cases are explained with the help of capacitor 1105 and the associated switch-array 1106 for clock period number $(n+1)$. One port of each capacitor in the array 1101 is permanently connected to the inverting input of amplifier 1103.

Case (1): An uncharged capacitor 1105 can be added to the active array $C_{ACC,TOT}(n)$ by configuring 1106 such that the second port is connected to the amplifier
10 output during the whole period number $(n+1)$. This causes a redistribution of the charges

and a thus a change in the voltage U_{ACC} , i.e., it changes from $\frac{Q_{ACC}(n)}{C_{ACC,TOT}(n)}$ to

$\frac{Q_{ACC}(n)}{C_{ACC,TOT}(n) + 2C_{ACC}}$, where $Q_{ACC}(n)$ is the charge in array 1105 at the end of the clock period number n and in the "sampling-section" of period number $(n+1)$. The magnitude of U_{ACC} is decreased in this case, since the overall capacitance

- 15 $C_{ACC,TOT}(n+1) = C_{ACC,TOT}(n) + 2C_{ACC}$ has been increased at a constant charge.

Case (2): Removing capacitor 1105 from the active array $C_{ACC,TOT}$ is achieved by switching 1106 such that the second port is connected to the reference voltage $V_{ref}/2$ during the whole period number $(n+1)$. Since this potential is equal the virtual potential of the inverting input of amplifier 1103, the amplifier forces the output to change its

- 20 potential from $\frac{Q_{ACC}(n)}{C_{ACC,TOT}(n)}$ to $\frac{Q_{ACC}(n)}{C_{ACC,TOT}(n) - 2C_{ACC}}$. As above, $Q_{ACC}(n)$ is the charge in array 1105 at the end of the clock period number n and in the "sampling-section" of period number $(n+1)$. The magnitude of U_{ACC} is increased in this case, since the overall capacitance $C_{ACC,TOT}(n+1) = C_{ACC,TOT}(n) - 2C_{ACC}$ has been decreased at a constant charge.

- 25 In various embodiments of the invention, the adaptation of $C_{ACC,TOT}(n)$ is achieved based, at least in part, on digital signal $c_0(n)$. An example of an adaptation scheme is summarized in Table 2 for an 8-bit signal $c_0(n)$. Here, the first non-zero bit within $c_0(n)$ is directly used to define $C_{ACC,TOT}(n)$.

Table 2: Example of an adaptation scheme for Q-switching

Bit pattern $c_0(n)$	Overall capacitance $C_{ACC,TOT}(n)$
[1 x x x x x x]	$128C_{ACC}$
[0 1 x x x x x]	$64C_{ACC}$
[0 0 1 x x x x]	$32C_{ACC}$
[0 0 0 1 x x x]	$16C_{ACC}$
[0 0 0 0 1 x x]	$8C_{ACC}$
[0 0 0 0 0 1 x]	$4C_{ACC}$
[0 0 0 0 0 0 1 x]	$2C_{ACC}$
[0 0 0 0 0 0 0 1]	C_{ACC}

Note that the exact value of $C_{ACC,TOT}(n)$ is not critically important. In various
 5 embodiments of the invention, the adaptation algorithm ensures that (1) on average, the
 magnitude of voltage U_{ACC} is maximized, but the instantaneous value of U_{ACC} does not
 exceed specified limits at no time instant, and (2) the switching between different
 configurations of 1101 is performed without any significant loss of charge, and preferably
 no loss of charge. Loss of charge in 1101 results in accumulation errors and thus reduces
 10 the system performance.

The above-described manner for reducing the dynamic range of the comparator
 input signal may be applicable to a wide variety of adaptive and non-adaptive Σ - Δ
 modulators, and is not limited to an adaptive Σ - Δ modulator in which the adaptation stage
 tends to keep the instantaneous magnitude of the analog feedback signal $z(n)$ within the
 15 first range $[-a, +a]$ and greater than the analog input signal's $x(n)$ instantaneous
 magnitude.

Alternative embodiments of the invention may be implemented as a computer
 program product for use with a computer system. Such implementation may include a
 series of computer instructions fixed either on a tangible medium, such as a computer
 20 readable media (*e.g.*, a diskette, CD-ROM, ROM, or fixed disk), or fixed in a computer
 data signal embodied in a carrier wave that is transmittable to a computer system via a
 modem or other interface device, such as a communications adapter connected to a
 network over a medium. The medium may be either a tangible medium (*e.g.*, optical or
 analog communications lines) or a medium implemented with wireless techniques (*e.g.*,

microwave, infrared or other transmission techniques). The series of computer instructions embodies all or part of the functionality previously described herein with respect to the system. Those skilled in the art should appreciate that such computer instructions can be written in a number of programming languages for use with many computer architectures or operating systems. Furthermore, such instructions may be stored in any memory device, such as semiconductor, magnetic, optical or other memory devices, and may be transmitted using any communications technology, such as optical, infrared, microwave, or other transmission technologies. It is expected that such a computer program product may be distributed as a removable medium with accompanying printed or electronic documentation (*e.g.*, shrink wrapped software), preloaded with a computer system (*e.g.*, on system ROM or fixed disk), or distributed from a server or electronic bulletin board over the network (*e.g.*, the Internet or World Wide Web).

Although various exemplary embodiments of the invention have been disclosed, it should be apparent to those skilled in the art that various changes and modifications can be made which will achieve some of the advantages of the invention without departing from the true scope of the invention. These and other obvious modifications are intended to be covered by the appended claims.